Disproving $\forall x \in S, P(x)$ Statements

Find an $x \in S$ such that $P(x)$ is false.

Called a counterexample

Conjecture: For every $n \in \mathbb{Z}$, $n^2 - n + 11$ is prime.

n	0	1	-1	2	-2	3	-3	4
#	11	11	13	13	17	17	23	23

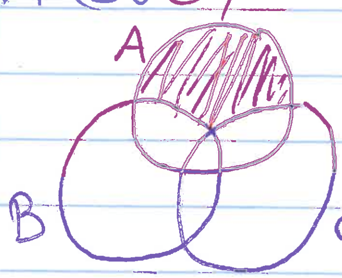
Since these seem to come in pairs, let's stick with positive #'s

n	5	6	7	8	9	10	11
#	31	41	53	67	83	101	121

So $n = 11$ produces a counterexample.

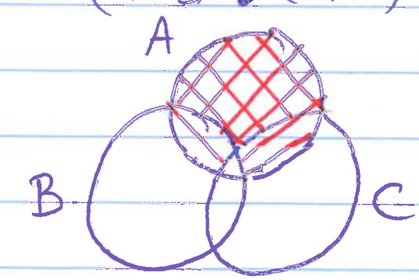
Conjecture: $A - (B \cup C) = (A - B) \cup (A - C)$

$A - (B \cup C)$



$\neq$

$(A-B) \cup (A-C)$



Find a counterexample

②

$A = \{1, 3, 5\}$

$B = \{3, 4\}$

$C = \{5, 6\}$

$B \cup C = \{3, 4, 5, 6\}$

$A - (B \cup C) = \{1\}$

$A - B = \{1, 5\}$

$A - C = \{1, 3\}$

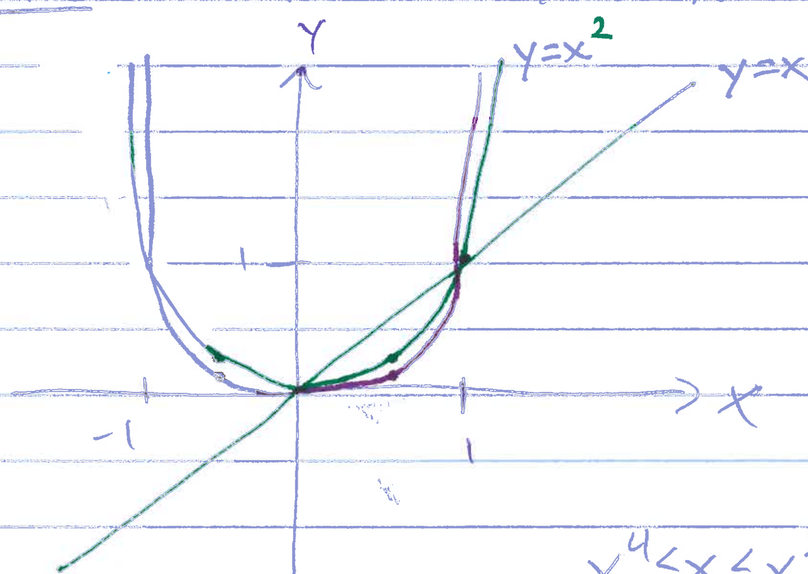
$(A-B) \cup (A-C) = \{1, 3, 5\} \neq A - (B \cup C)$

Disproving $\exists x \in S, P(x)$ Statements

To disprove this we have to prove

$$\forall x \in S, \sim P(x)$$

Conjecture $\exists x \in \mathbb{R}$ such that $x^4 < x < x^2$.



$x^4 < x < x^2$ is never true in the graph.

So, we have to show that $\forall x \in \mathbb{R}$, $x^4 < x < x^2$ is false